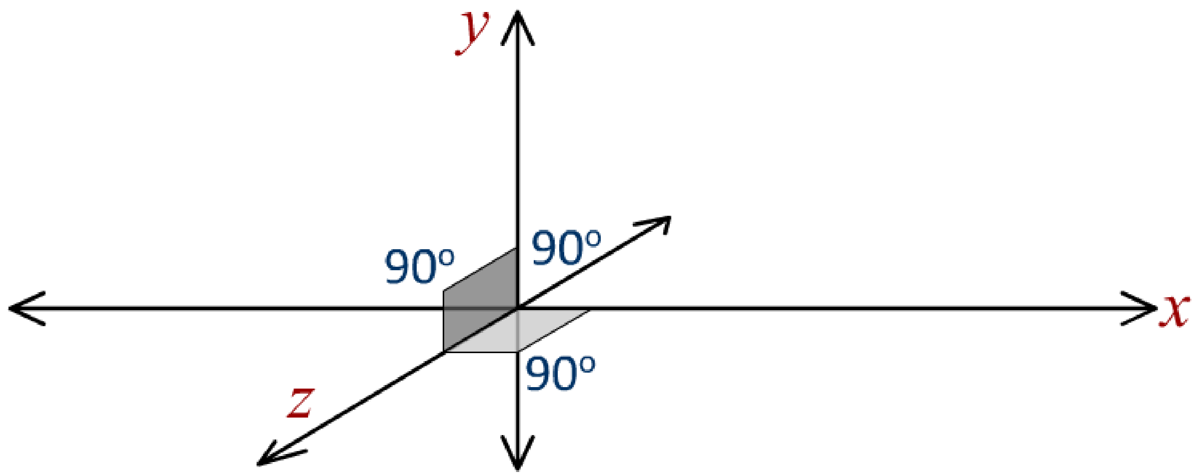
Equations of motion

Position

Position of a body refers to its location w.r.t. to reference. A reference frame is a combination of mutually perpendicular axes intersecting at a common point called origin.



Number of coordinates required to uniquely locate the body depends on the kind of motion.

Motion	No. of coordinates	Examples
One dimensional	One (x)	Freely falling body
Two dimensional	Two (x, y)	Projectile
Three dimensional	Three (x, y, z)	A bird in flight

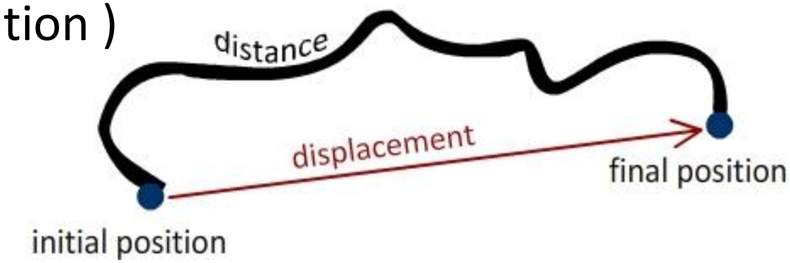
Motion in a straight line

Distance

- It is the length of the path followed by body as it moves from its initial to its final position.
- It can be either zero (for a stationary body) or non-zero for a body in motion
- It is a scalar quantity
- SI unit is m
- It is measured by the odometer in our vehicles

Displacement

- It is given by the length of the shortest line drawn from the initial position to the final position a body undergoing motion.
- It may be zero, positive or negative depending on the motion of the body
- It is a vector quantity (from initial to final position)
- SI unit is m
- It is calculated based on our observation.



Motion in a straight line

- Velocity : It is defined as the rate of change of displacement w.r.t. time
- It is vector quantity
- SI unit of velocity is ms^{-1}
- Average velocity is given by displacement of the body in the given time interval divided by time.

$$v_{\text{avg}} = \frac{\Delta S}{\Delta t}$$

- Instantaneous velocity is given by the rate of displacement w.r.t. time

$$v_{\text{inst}} = \lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t}$$

Motion in a straight line

- Acceleration : It is defined as the rate of change of velocity w.r.t. time
- It is vector quantity
- SI unit of acceleration is ms^{-2}
- Average acceleration is given by total change in velocity divided by the time interval

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}$$

- Instantaneous acceleration is given by the rate of change of velocity w.r.t. time

$$a_{\text{inst}} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

Motion in a straight line

- Slope of $S-t$ plot gives the rate at which displacement occurs.
- For a certain interval of time, the slope gives average velocity (and in the process the finer details of any increase/decrease are lost)
- Choosing a very small interval of time gives the instantaneous velocity.
- In choosing a very small interval, slope of graph at that point becomes the tangent to the curve.

- Slope of $v-t$ plot gives the rate at which the velocity changes
- For a certain interval of time, the slope gives average acceleration (and in the process the finer details of any increase/decrease are lost)
- Choosing a very small interval of time gives the instantaneous acceleration.
- In choosing a very small interval, slope of graph at that point becomes the tangent to the curve.

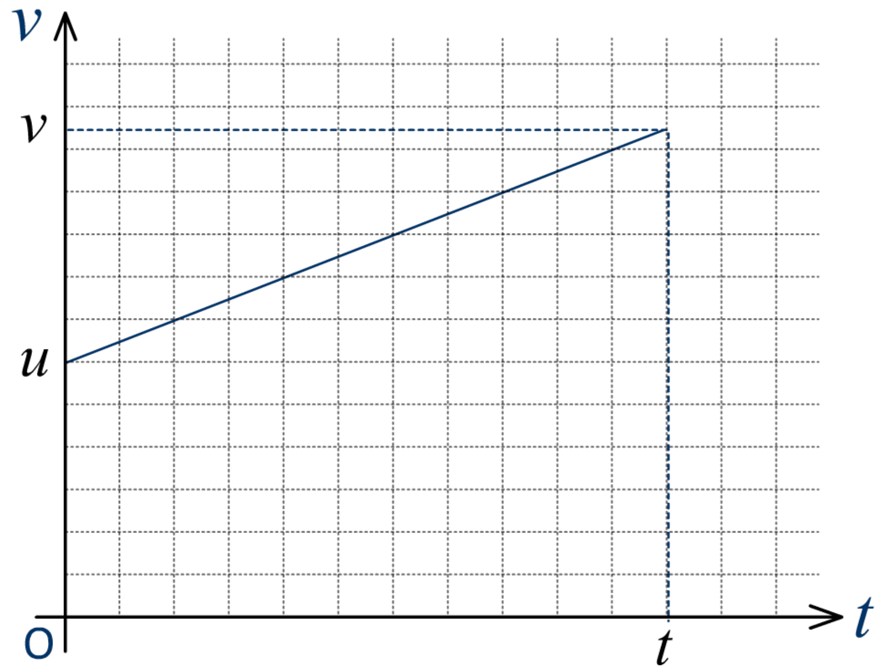
- Area of acceleration versus time plot gives velocity
- Area of velocity versus time plot gives displacement

Equations of motion

(relations between velocities, acceleration, displacement and time)

Consider a body having initial velocity u and moving uniform acceleration a to time t.

Velocity versus time plot for such a motion is as shown in the figure.



Slope of the plot is given by

$$\text{slope} = \frac{\Delta y}{\Delta x}$$

$$\text{slope} = \frac{\Delta v}{\Delta t}$$

$$\text{slope} = \frac{v - u}{t - 0}$$

$$\text{slope} = \frac{v - u}{t}$$

The rate of change of velocity is called acceleration, therefore

$$a = \frac{v - u}{t}$$

$$\boxed{v = u + at} \quad (1)$$

Equations of motion

(relations between velocities, acceleration, displacement and time)

Area under the curve of is given by

$$\text{Area} = A_1 + A_2 \quad \text{(i)}$$

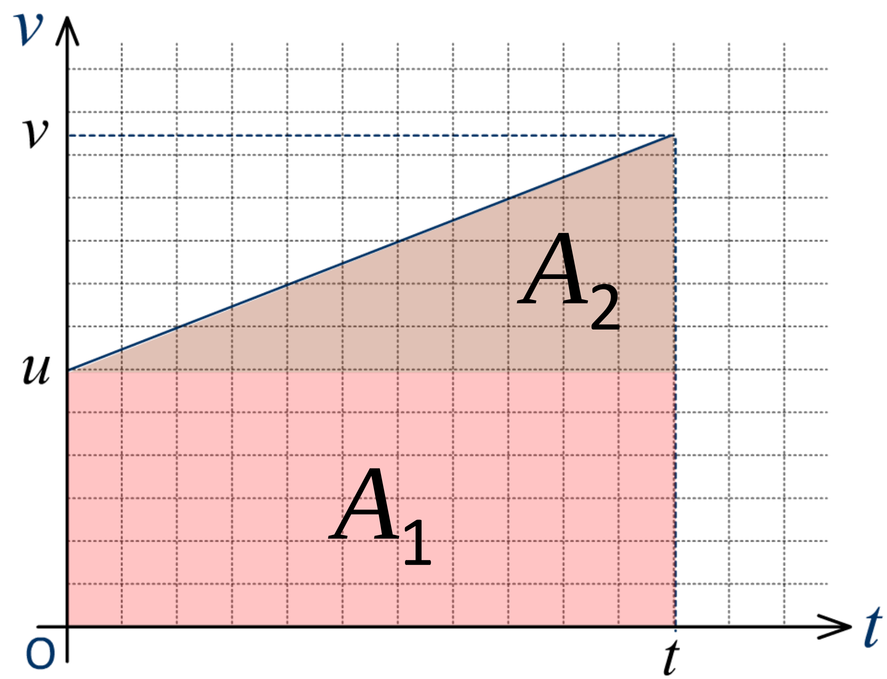
$$A_1 = u \times t \quad \text{(ii)}$$

$$A_2 = \frac{1}{2} \text{ base} \times \text{ alt}$$

$$A_2 = \frac{1}{2} t \times (v - u) \quad \text{(iii)}$$

Substituting (iii) and (ii) in (i) we get

$$S = ut + \frac{1}{2} t \times (v - u) \quad \text{(iv)}$$



Using the relation

$$v = u + at \quad \Rightarrow v - u = at$$

$$S = ut + \frac{1}{2} t \times at$$

$$\boxed{S = ut + \frac{1}{2} at^2} \quad (2)$$

Equations of motion

(relations between velocities, acceleration, displacement and time)

Using the relation of velocity as a function of time and displacement as a function of time we can obtain the relation between velocity an displacement

$$\boxed{S = ut + \frac{1}{2}at^2} \quad (2)$$

$$\boxed{v = u + at} \quad (1)$$

$$t = \frac{v-u}{a}$$

Substituting this in equation (2) we get

$$S = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2$$

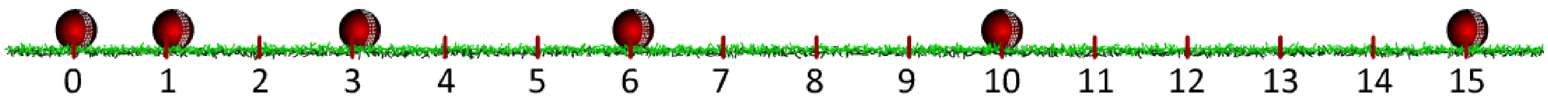
$$S = \left(\frac{v-u}{a}\right)\left(u + \frac{1}{2}a\left(\frac{v-u}{a}\right)\right)$$

$$S = \left(\frac{v-u}{a}\right)\left(u + \frac{v-u}{2}\right)$$

$$S = \left(\frac{v-u}{a}\right)\left(\frac{v+u}{2}\right)$$

$$\boxed{v^2 - u^2 = 2aS} \quad (3)$$

Displacement in the n^{th} second of motion



Consider a body having an initial velocity (u) and moving with uniform acceleration (a) . Its displacement, as a function of time, is given by the relation

$$S = ut + \frac{1}{2}at^2$$

Displacement in the n^{th} second is

$$S(n) = un + \frac{1}{2}an^2$$

Displacement in the (n -1) th second is

$$S(n-1) = u(n-1) + \frac{1}{2}a(n-1)^2$$

Subtracting (ii) from (i) we get

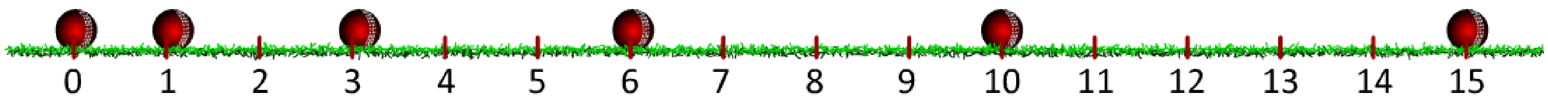
$$S_n = un + \frac{1}{2}an^2 - u(n-1) - \frac{1}{2}a(n-1)^2$$

$$S_n = un + \frac{1}{2}an^2 - un + u - \frac{an^2}{2} + an - \frac{a}{2}$$

$$\boxed{S_n = u + a\left(n - \frac{1}{2}\right)}$$

Motion in a straight line

Average velocity



Average velocity of a body is given by

$$v_{avg} = \frac{S_{total}}{t_{total}}$$

Total displacement of the body is given by

$$S = ut + \frac{1}{2}at^2$$

$$v_{avg} = \frac{ut + \frac{1}{2}at^2}{t}$$

$$v_{avg} = u + \frac{1}{2}at$$

Using the relation for velocity

$$v = u + at$$

$$\Rightarrow at = v - u$$

$$v_{avg} = u + \frac{1}{2}(v - u)$$

$$\boxed{v_{avg} = \frac{v + u}{2}}$$

* Applicable only for motion with constant acceleration